On the Informed Principal Model with Common Values

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Motivation (I)

- Conventional mechanism design theory assumes that the party that designs a mechanism (i.e., the principal) has no private (payoff-relevant) information
- However, in many occasions the principal may have private information. For instance:
  - **Procurement:** The government may have superior information about the cost of a project than potential constructors
  - **Vertical contracting:** An upstream manufacturer may have more detailed information at the time of contracting about market characteristics, e.g., the market demand, than a downstream retailer
  - **Informed seller:** A seller of an indivisible object may have superior information about the quality of the object than potential buyers

Motivation (II)

- In all these occasions, the principal has private information that affects the value of trade
- When offering the mechanism the principal may reveal part of this information and therefore the design of the mechanism itself becomes subtle
- Question: What mechanism will the principal select?
- Myerson (1983) took an axiomatic approach to characterise a reasonable solution
- Maskin and Tirole (1990, 1992) (MT) took a non-cooperative approach to characterise mechanisms that can result as equilibria in a three stage game (mechanism proposal/acceptance-rejection/mechanism execution)

Contribution

- I identify some fundamental properties of the Rothschild-Stiglitz-Wilson allocation, i.e., the undominated allocation within the set of incentive compatible and individually rational for the agent type by type allocations
- Based on these properties, I constuct a more robust, and perhaps simpler, proof of Theorem 1 (the main result) of Maskin and Tirole (1992)
- I make a distinction between simple mechanisms (in which the agent makes no announcement, e.g., DRMs) and general mechanisms
- I provide a simple example to highlight why such distinction is important
- I provide a more general condition than the no-tangency condition provided in Maskin and Tirole (1992) that allows for the complete characterisation of the set of equilibrium allocations

The Model with Unilateral Private Information

- Two players: a principal (P) and an agent (A)
- Type of principal $i = 1, ..., n$, $n \geq 2$, is her private information
- The set of actions is $X \subset \mathbb{R}^K$, $K \geq 1$ (compact)
- Prior beliefs: $\Pi^{\cdot} = (\Pi^i)_i$, where $\Pi^i > 0$ for every i
- Payoffs: $V^i(x)$ and $U^i(x)$ (continuous) for the principal and the agent respectively
- The two players wish to select a (potentially random) contractible action from $\mathcal{M} = \Delta(X)$
- Let $V^i(\mu) = \int V^i(x) d\mu$ and $U^i(\mu) = \int U^i(x) d\mu$

Allocations - IC, Dominance, IE

- *Allocation*: $\mu^{\cdot} = (\mu^i)_i$

Definition

An allocation $\mu^{\cdot}$ is incentive compatible (IC) if $U^i(\mu^i) \geq U^i(\mu^j)$ for every i, j

Definition

An IC allocation $\mu^{\cdot}$ dominates an IC allocation $\mu^{\cdot}$ if $V^i(\mu^i) \geq V^i(\bar{\mu}^i)$ for every i with the inequality being strict for at least one i

Definition

*An allocation $\bar{\mu}^{\cdot}$ is interim efficient (IE) relative to beliefs $\bar{\Pi}^{\cdot}$ (not necessarily the prior beliefs) if (i) it is IC, and (ii) there exists no allocation $\mu^{\cdot} \neq \bar{\mu}^{\cdot}$ that is IC, satisfies $\sum \bar{\Pi}^i U^i(\mu^i) \geq \sum \bar{\Pi}^i U^i(\bar{\mu}^i)$, and dominates $\bar{\mu}^{\cdot}$.**

*An allocation is weakly interim efficient (WIE) (or IE type by type) if we substitute $\sum \bar{\Pi}^i U^i(\mu^i) \geq \sum \bar{\Pi}^i U^i(\bar{\mu}^i)$ with $U^i(\mu^i) \geq U^i(\bar{\mu}^i)$ for every i.

Reservation Allocation - IR

- *Reservation Allocation*: $\mu_0^{\cdot}$; it can be regarded as either an outside option or a prior contract that binds the two players and they wish to renegotiate.

Definition

*An IC allocation $\mu^{\cdot}$ is individually rational (IR) relative to beliefs $\bar{\Pi}^{\cdot}$ (not necessarily the prior beliefs) if $\sum \bar{\Pi}^i U^i(\mu^i) \geq \sum \bar{\Pi}^i U^i(\mu_0^i)$.**

* An allocation $\mu^{\cdot}$ is IR type by type if we substitute $\sum \bar{\Pi}^i U^i(\mu^i) \geq \sum \bar{\Pi}^i U^i(\mu_0^i)$ with $U^i(\mu^i) \geq U^i(\mu_0^i)$ for every i.

The Rothschild-Stiglitz-Wilson (RSW) Allocation

Definition

An allocation $\mu^{\cdot}$ is an RSW allocation (relative to the reservation allocation $\mu_0^{\cdot}$) if (i) it is IC and IR type by type, and (ii) there exists no allocation $\mu^{\cdot} \neq \bar{\mu}^{\cdot}$ that is IC, IR type by type, and dominates $\mu^{\cdot}$.

The Rothschild-Stiglitz-Wilson (RSW) Allocation

Lemma

Every RSW allocation is payoff equivalent for the principal.

Proof.

Suppose that $\hat{\mu}_1^{\cdot}(\mu_0^{\cdot})$ and $\hat{\mu}_2^{\cdot}(\mu_0^{\cdot})$ are RSW allocations, where $\hat{\mu}_1^{\cdot}(\mu_0^{\cdot}) \neq \hat{\mu}_2^{\cdot}(\mu_0^{\cdot})$ and $V^i(\hat{\mu}_1^i(\mu_0^{\cdot})) \neq V^i(\hat{\mu}_2^i(\mu_0^{\cdot}))$ for some i. Let $I_1 = \{i : V^i(\hat{\mu}_1^i(\mu_0^{\cdot})) \geq V^i(\hat{\mu}_2^i(\mu_0^{\cdot}))\}$ and $I_2 = \{i : V^i(\hat{\mu}_1^i(\mu_0^{\cdot})) < V^i(\hat{\mu}_2^i(\mu_0^{\cdot}))\}$. Because $\hat{\mu}_1^{\cdot}(\mu_0^{\cdot})$ and $\hat{\mu}_2^{\cdot}(\mu_0^{\cdot})$ are IC and IR type by type, allocation $\tilde{\mu}^{\cdot}$, which maps each type from I_1 to her action in allocation $\hat{\mu}_1^{\cdot}(\mu_0^{\cdot})$ and each type from I_2 to her action in allocation $\hat{\mu}_2^{\cdot}(\mu_0^{\cdot})$, is also IC and IR type by type. Allocation $\tilde{\mu}^{\cdot}$ dominates both $\hat{\mu}_1^{\cdot}(\mu_0^{\cdot})$ and $\hat{\mu}_2^{\cdot}(\mu_0^{\cdot})$, which contradicts the definition of an RSW allocation. □

Mechanisms

- *Mechanism:* $m = (S, g)$, $S = S_P \times S_A$ (finite), $g : S \to \mathcal{M}$

Definition

For given beliefs $\bar{\Pi}^{\cdot}$, a Bayesian Nash equilibrium in mechanism m consists of a profile of strategies, one for each player, such that, conditional on the strategy of the other player, no player has a unilateral profitable deviation.

- Every equilibrium in mechanism m under beliefs $\bar{\Pi}^{\cdot}$ is associated with an ex post (expected) equilibriumpayoff profile $(\bar{V}^{\cdot}(m, \bar{\Pi}^{\cdot}), \bar{U}^{\cdot}(m, \bar{\Pi}^{\cdot}))$, where $\bar{V}^{\cdot}(m, \bar{\Pi}^{\cdot}) = (\bar{V}^{i}(m, \bar{\Pi}^{\cdot}))_i$ and $\bar{U}^{\cdot}(m, \bar{\Pi}^{\cdot}) = (\bar{U}^{i}(m, \bar{\Pi}^{\cdot}))_i$
- Simpler class of mechanisms is the class of *direct revelation mechanisms* (DRMs), in which the principal simply announces a type (not necessarily the true type), and the agent makes no announcement.

Properties of the RSW Allocation - General Mechanisms

Proposition

Suppose that $\hat{\mu}^{\cdot}(\mu_0^{\cdot})$ *(i.e., the RSW allocation) is IE relative to beliefs* $\hat{\Pi}^{\cdot}$ *(not necessarily the prior beliefs), where* $\hat{\Pi}^i > 0$ *for every* i*; then, for every mechanism* $m \neq \hat{\mu}^{\cdot}(\mu_0^{\cdot})$ *and subset of types* $I \subseteq \{1, ..., n\}$*, there exist beliefs* $\bar{\Pi}^{\cdot}$ *such that in every equilibrium of* m *under* $\bar{\Pi}^{\cdot}$ *with an associated equilbrium payoff profile* $(\bar{V}^{\cdot}(m, \bar{\Pi}^{\cdot}), \bar{U}^{\cdot}(m, \bar{\Pi}^{\cdot}))$*, either*

$$\bar{V}^i(m, \bar{\Pi}^{\cdot}) \leq V^i(\hat{\mu}^i(\mu_0^{\cdot})) \quad \textit{for every} \quad i \in I \tag{1}$$

or

$$\sum \bar{\Pi}^i \bar{U}^i(m, \bar{\Pi}^{\cdot}) < \sum \bar{\Pi}^i U^i(\mu_0^i) \tag{2}$$

Proof of Proposition 1

Suppose that there exists $m \neq \hat{\mu}^{\cdot}(\mu_0^{\cdot})$ and $I \subseteq \{1, ..., n\}$ such that for every $\bar{\Pi}^{\cdot}$, there exists an equilibrium with an associated equilibrium payoff profile $(\bar{V}^{\cdot}(m, \bar{\Pi}^{\cdot}), \bar{U}^{\cdot}(m, \bar{\Pi}^{\cdot}))$, such that: (i) $\bar{V}^i(m, \bar{\Pi}^{\cdot}) > V^i(\hat{\mu}^i(\mu_0^{\cdot}))$ for every $i \in I$, and $\bar{V}^i(m, \bar{\Pi}^{\cdot}) \leq V^i(\hat{\mu}^i(\mu_0^{\cdot}))$ for every $i \notin I$, and, (ii) $\sum \bar{\Pi}^i \bar{U}^i(m, \bar{\Pi}^{\cdot}) \geq \sum \bar{\Pi}^i U^i(\mu_0^i)$
Consider $\Pi_I^{\cdot}$, where $\Pi_I^i = \hat{\Pi}^i / \sum_{j \in I} \hat{\Pi}^j$ for every $i \in I$, and $\Pi_I^i = 0$ for every $i \notin I$.
Construct $\tilde{\mu}^{\cdot}$, where $V^i(\tilde{\mu}^i) = \bar{V}^i(m, \Pi_I^{\cdot})$ for every $i \in I$ and $\tilde{\mu}^i = \hat{\mu}^i(\mu_0^{\cdot})$ for every $i \notin I$. $\tilde{\mu}^{\cdot}$ is IC because of (i) and $\hat{\mu}^{\cdot}(\mu_0^{\cdot})$ is IC by definition. Moreover,

$$\sum \hat{\Pi}^i U^i(\tilde{\mu}^i) = \sum_{i \in I} \hat{\Pi}^i \bar{U}^i(m, \Pi_I^{\cdot}) + \sum_{i \notin I} \hat{\Pi}^i U^i(\hat{\mu}^i(\mu_0^{\cdot})) \geq \sum \hat{\Pi}^i U^i(\mu_0^i) \quad (3)$$

because $\sum_{i \in I} \frac{\hat{\Pi}^i}{\sum_{j \in I} \hat{\Pi}^j} \bar{U}^i(m, \Pi_I^{\cdot}) \geq \sum_{i \in I} \frac{\hat{\Pi}^i}{\sum_{j \in I} \hat{\Pi}^j} U^i(\mu_0^i)$ from (ii) above and due to the fact that the RSW allocation is IR type by type. $\tilde{\mu}^{\cdot}$ dominates $\hat{\mu}^i(\mu_0^{\cdot})$ and is IR relative to beliefs $\hat{\Pi}^{\cdot}$; a contradiction.

Properties of the RSW Allocation - Simple Mechanisms

Proposition

Suppose that the principal is restricted to offering only DRMs; then, for every IC allocation $\mu^{\cdot} \neq \hat{\mu}^{\cdot}(\mu_0^{\cdot})$ *(where* $\hat{\mu}^{\cdot}(\mu_0^{\cdot})$ *is the RSW allocation) and subset of types* $I \subseteq \{1, ..., n\}$*, there exist beliefs* $\bar{\Pi}^{\cdot}$ *such that either*

$$V^i(\mu^i) \leq V^i(\hat{\mu}^i(\mu_0^{\cdot})) \quad \text{for every} \quad i \in I \tag{4}$$

or

$$\sum \bar{\Pi}^i U^i(\mu^i) < \sum \bar{\Pi}^i U^i(\mu_0^i) \tag{5}$$

Proof of Proposition 2

Suppose that there exists an IC allocation $\mu^{\cdot} \neq \hat{\mu}^{\cdot}(\mu_0^{\cdot})$ and $I \subseteq \{1, ..., n\}$ such that for every $\bar{\Pi}^{\cdot}$: (i) $V^i(\mu^i) > V^i(\hat{\mu}^i(\mu_0^{\cdot}))$ for every $i \in I$ and $V^i(\mu^i) \leq V^i(\hat{\mu}^i(\mu_0^{\cdot}))$ for every $i \notin I$, and (ii) $\sum \bar{\Pi}^i U^i(\mu^i) \geq \sum \bar{\Pi}^i U^i(\mu_0^i)$. Consider $\tilde{\mu}^{\cdot}$, where

$$\tilde{\mu}^i = \begin{cases} \mu^i, & \text{if } \; i \in I \\ \hat{\mu}^i(\mu_0^{\cdot}), & \text{otherwise} \end{cases}$$

This allocation is IC because $\mu^{\cdot}$ and $\hat{\mu}^{\cdot}(\mu_0^{\cdot})$ are IC and (i) above. Moreover, $\tilde{\mu}^{\cdot}$ is IR type by type because $\hat{\mu}^{\cdot}(\mu_0^{\cdot})$ and $\mu^{\cdot}$ are IR type by type (by the definition of the RSW allocation and (ii) above). Therefore, $\mu^{\cdot}$ dominates $\hat{\mu}^{\cdot}(\mu_0^{\cdot})$ and is IR type by type, which contradicts $\hat{\mu}^{\cdot}(\mu_0^{\cdot})$ being an RSW allocation.

Example 1

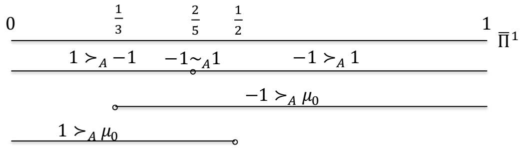
- $n = 2$, $\Pi^1 = \Pi^2 = 1/2$ and $\mathcal{M} = \{-1, 0, 1\}$

μ	$U^1(\mu)$	$U^2(\mu)$	$V^1(\mu)$	$V^2(\mu)$
-1	2	-1	2	2
0	0	0	1	1
1	-1	1	3	3
μ_0	0	0	0	0

Table: Payoffs

- IC allocations: $\{(-1,-1), (0,0), (1,1)\}$
- RSW allocation: $\hat{\mu}^{\cdot}(\mu_0^{\cdot}) = (0, 0)$

Example 1 cont'd



- RSW is not IE relative to any non-degenerate beliefs
- Consider mechanism $m^d = (S^d, g^d)$, where $S_P^d = \emptyset$, $S_A^d = \{-1, 1\}$ and $g^d(s) = s$ for every $s \in S_A^d$
- For DRM $(-1, -1)$ and $\bar{\Pi}^1 < 1/3$, $\sum \bar{\Pi}^i U^i(-1) < 0$; for DRM $(1, 1)$ and $\bar{\Pi}^1 > 1/2$, $\sum \bar{\Pi}^i U^i(1) < 0$

The Extensive-Form Game

- As in MT let the two players play the following game
  1. The principal proposes a mechanism
  2. The agent accepts of rejects
  3. If the agent rejects, the reservation action is in effect. If he accepts, the two players play the mechanism proposed by the principal
- Strategies of players
- Update of beliefs
- *Perfect Bayesian Equilibrium*
- **Inscrutability Principle (Myerson 1983)**: There is no loss of generality in concentrating on equilibria in which all types offer the same mechanism

Equilibrium Allocations

Suppose that X is convex and a type i indifference curve is nowhere tangent to a type j indifference curve

Theorem (MT92)

Suppose that $\hat{\mu}^{\cdot}(\mu_0^{\cdot})$ (i.e., the RSW allocation) is IE relative to beliefs $\hat{\Pi}^{\cdot}$ (not necessarily the prior beliefs), where $\hat{\Pi}^i > 0$ for every i; then, an allocation $\bar{\mu}^{\cdot}$ is an equilibrium allocation of the three-stage game if and only if it is IC and satisfies

$$V^i(\bar{\mu}^i) \geq V^i(\hat{\mu}^i(\mu_0^{\cdot})) \;\forall\; i \tag{6}$$

$$\sum \Pi^i U^i(\bar{\mu}^i) \geq \sum \Pi^i U^i(\mu_0^i) \tag{7}$$

Sketch of Proof of Theorem 1

* For the "if" part:

- The sets of mechanisms can be partitioned in two subsets: Set A includes those mechanisms that are IR for the agent relative to all possible beliefs; Set A^c includes every other feasible mechanism
- Suppose that the principal offers $\bar{\mu}^{\cdot}$; an IC allocation that satisfies (8) and (7)
- To construct an equilibrium one needs to assign beliefs to every feasible mechanism and specify sequentially rational strategies

Sketch of Proof of Theorem 1 cont'd

- For every mechanism in A, Proposition 1 assures that there exist beliefs such that all types are worse off relative to the RSW allocation
- For every mechanism in A^c, assign beliefs such that the mechanism is not IR for the agent
- Construct sequentially rational strategies for the principal and the agent given these beliefs
- Mission accomplished!

* For the "only if" part key is the assumption that the indifference curves of different types are nowhere tangent.

Robustness

- Maskin and Tirole (1992) show that for every mechanism different from the on-the-equilibrium path mechanism there exist beliefs and **an** equilibrium (i.e., continuation of the game) such that every type is worse off
- The proof proposed here shows that for every mechanism different from the on-the-equilibrium path mechanism there exist beliefs such that in **every** equilibrium (i.e., continuation of the game) every type is worse off

A More General Sufficient Condition for the "Only If"

- The "nowhere no-tangency" condition might be too strong
- Is there a milder condition that allows for the complete characterisation of the set of equilibrium allocations?
- Two further definitions

Definition

An allocation $\mu^{\cdot}$ is strictly incentive compatible (SIC) if $V^i(\mu^i) > V^i(\mu^j)$ for every i, j.

Definition

An IC allocation $\mu^{\cdot}$ is strictly individually rational (SIR) type by type if $U^i(\mu^i) > U^i(\mu_0^i)$.

A More General Sufficient Condition for the "Only If"

Proposition

Suppose that there exists a sequence of SIC and SIR type by type allocations $\{\mu^{\cdot}_p\}_{p=1}^{\infty}$, *i.e.,* $V^i(\mu^i_p) > V^i(\mu^j_p)$ *for every* i, j, p *and* $U^i(\mu^i_p) > U^i(\mu^i_0)$ *for every* i, p, *such that* $\{V^i(\mu^i_p)\}_{p=1}^{\infty}$ *converges to* $V^i(\hat{\mu}^i(\mu^{\cdot}_0))$ *for every* i; *then every equilibrium allocation of the three-stage game* $\bar{\mu}^{\cdot}$ *is such that*

$$V^i(\bar{\mu}^i) \geq V^i(\hat{\mu}^i(\mu^{\cdot}_0)) \ \forall \ i \tag{8}$$

Proof of Proposition 3

Consider an IC allocation $\bar{\mu}^{\cdot}$ in which for some type j, $V^j(\bar{\mu}^j) < V^j(\hat{\mu}^j(\mu_0^{\cdot}))$. Let $V^j(\hat{\mu}^j(\mu_0^{\cdot})) - V^j(\bar{\mu}^j) = \delta > 0$. The following lemma facilitates the proof.

Lemma

There exists p_δ such that $V^j(\mu_p^j) \geq V^j(\bar{\mu}^j)$ for every $p \geq p_\delta$.

Proof of Lemma. Because there exists a sequence $\{\mu_p^{\cdot}\}_{p=1}^{\infty}$ such that $\{V^i(\mu_p^i)\}_{p=1}^{\infty}$ converges to $V^i(\hat{\mu}^i(\mu_0^{\cdot}))$ for every i, there exists p_δ such that $|V^j(\mu_p^j) - V^j(\hat{\mu}^j(\mu_0^{\cdot}))| < \delta$ for every $p \geq p_\delta$. Suppose that $V^j(\mu_p^j) > V^j(\hat{\mu}^j(\mu_0^{\cdot}))$ for some $p \geq p_\delta$. Consider $\tilde{\mu}^{\cdot}$, where $\tilde{\mu}^i = \mu_p^i$ for $i = j$ and $\tilde{\mu}^i = \hat{\mu}^i(\mu_0^{\cdot})$ for $i \neq j$. Allocation $\tilde{\mu}^{\cdot}$ is IC and IR type by type which contradicts the definition of the RSW allocation. Therefore, $V^j(\mu_p^j) \leq V^j(\hat{\mu}^j(\mu_0^{\cdot}))$ for every $p \geq p_\delta$. Then, $V^j(\hat{\mu}^j(\mu_0^{\cdot})) - V^j(\mu_p^j) < \delta$ and hence $V^j(\hat{\mu}^j(\mu_0^{\cdot})) - V^j(\mu_p^j) < V^j(\hat{\mu}^j(\mu_0^{\cdot})) - V^j(\bar{\mu}^j)$, which is equivalent to $V^j(\mu_p^j) > V^j(\bar{\mu}^j)$.

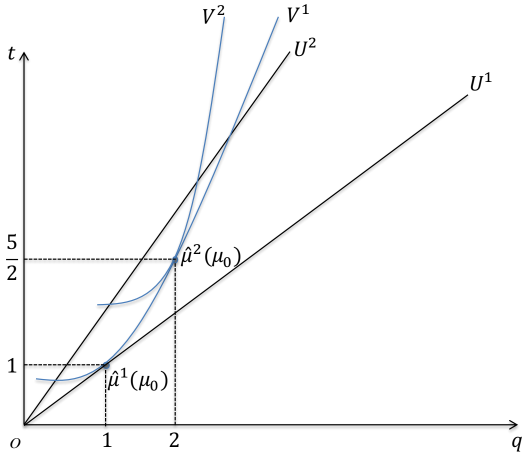
Proof of Proposition 3 cont'd

Consider mechanism $\mu^{\cdot}_{p'_\delta}$, where $p'_\delta \geq p_\delta$. Because this mechanism is SIC and SIR type by type, it provides the agent with a payoff strictly greater than the payoff he can obtain in the reservation allocation regardless of his beliefs. Therefore, if this mechanism is proposed by type j, it should be accepted by the agent; otherwise the equilibrium fails to be sequentially rational. Type j can achieve a higher payoff by proposing mechanism $\mu^{\cdot}_{p'_\delta}$ than by proposing $\bar{\mu}^{\cdot}$, which means that allocation $\bar{\mu}^{\cdot}$ cannot constitute an equilibrium allocation.

Example 2

- n=2, $K = 2$
- $V^1(t,q) = t - q^2/2$, $V^2(t,q) = t - 2q(1+q)/5$
- $MRS^1_{t,q} = -\frac{V^1_q}{V^1_t} = q$, $MRS^2_{t,q} = -\frac{V^2_q}{V^2_t} = 2(1+2q)/5$
- Indifference curves are tangent at $q = 2$, $MRS^1_{t,q} > MRS^2_{t,q}$ if $q > 2$ and $MRS^1_{t,q} < MRS^2_{t,q}$ if $q < 2$
- RSW allocation is $\big((1,1),(2,5/2)\big)$

Example 2 cont'd



Example 2 cont'd

- Consider $\mu_p^1 = (1 - 1/p, 1)$ and $\mu_p^2 = (5/2 - 2/p, 2)$

$$V^1(t_p^1, q_p^1) = 1/2 - 1/p > 1/2 - 2/p = V^1(t_p^2, q_p^2) \text{ for every } p$$

$$V^2(t_p^2, q_p^2) = -51/30 - 2/p > 1/5 - 1/p = V^2(t_p^1, q_p^1) \text{ for every } p$$

$$U^1(t_p^1, q_p^1) = 1/p > 0, U^2(t_p^2, q_p^2) = 3/2 + 2/p > 0 \text{ for every } p$$

- Hence $\mu_p^{\cdot}$ is SIC and SIR for every p and

$$\{V^i(t_p^i, q_p^i)\}_p \rightarrow V^i(\hat{\mu}^i(\mu_0^{\cdot})) \text{ for every } i$$

Take-aways

- In general environments, restriction to DRMs restricts the set of profitable deviations and therefore allows one to establish the existence of equilibrium
- However, this is with loss of generality
- The revelation principle holds (i.e., restriction to DRMs is without loss of generality) iff the RSW allocation is IE relative to some non-degenerate beliefs

Future Work

- Better characterisation of environments that satisfy the sufficient condition that allows for the complete characterisation of the set of equilibrium allocations
- Better characterisation of environments in which the RSW allocation is IE relative to some non-degenerate beliefs
- Environments with bilateral private information
- Applications