Strategic Experimentation in an R&D Race

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Dynamic R&D with Learning

- R&D is a **dynamic (competitive) process**
- Firms decide whether or not to undertake R&D in a potentially infinite number of periods
- R&D is **uncertain** both regarding the time of completion as well as the viability of the project
  - Mansfield et al. (1971): Probability of technical completion of a project $\simeq$ 57%
  - Di Masi et al. (2003,2016): Pharmaceuticals $\simeq$ 20-25%
- **Learning** is an essential element: Firms (or research teams) learn from past and rivals
  - Superconductivity, software, The Human Genome Project, etc.
- This paper: A (tractable) two-stage R&D race with learning

The Model: Firms and R&D

- There is a potentially infinite number of periods: $t = 0, 1, ...$
- No discounting
- Two symmetric firms
- A novel product (e.g., drug) of social return R
- Two stages need to be completed: Reseach and Development ($s = r, d$)
- The first firm that completes the two stages acquires a patent and γR, where $\gamma \leq 1$
- **Comments**:
  - If one firm advances to D and the other does not, the lagging firm can catch up
  - However, it needs to pass also R (no perfect imitation)
  - Leader gains advantage

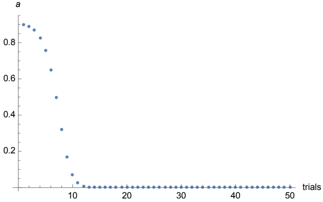
The Model: Bandits

- Every firm has a bandit technology in every stage (symmetric across firms)
- Bandit is armed in good or bad state
- Priors for good state: $\alpha_0^r \equiv \alpha_0 < 1$ and $\alpha_0^d = 1$
- If good probability to realise success $p_r < 1$ and $p_d < 1$
- Assumption that $\alpha_0^d = 1$ is only for simplicity
- Bandits are perfectly correlated: either both in bad or both in good
- **Effects**:
  - One firm succeeds, good news for the rival $\Rightarrow$ *Positive (information) effect* (known in Strategic Experimentation literature)
  - One firm succeeds, bad news for the rival $\Rightarrow$ *Negative (pre-emption) effect* (known in Patent Races literature)

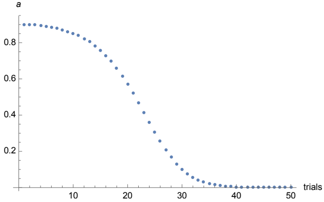
The Model: Beliefs

- At least one firm succeeds in R, belief jumps to one
- A failure brings bad news; after $\tau = 1, 2, \ldots$ unsuccessful trials:

$$\alpha_\tau = \frac{\alpha(1-p_r)^\tau}{(1-\alpha_0)+\alpha_0(1-p_r)^\tau}$$



(a) $a_0 = 0.9,\ p_r = 0.1$



(b) $a_0 = 0.9,\ p_r = 0.01$

Unique Symmetric Equilibrium: Notation

- Firms simultaneously decide whether to be active or not
- Cost of being active = 1 (for simplicity)
- **Important**: Firms decide to pay the cost every period (i.e., the can enter and exit anytime)
- Firms observe rivals actions and outcomes $\Rightarrow$ Game of complete information
- *Subgame Perfect Nash Equilibrium*
- $\bar{q}_{ss'}$: eq'um probability of a firm being active in s when rival is in s'
- $\bar{V}_{ss'}$: eq'um continuation payoff of a firm being active in s when rival is in s'
- Remarks: Symmetric eq'um and stationarity
- $\bar{q}^{\tau}_{rr}$: eq'um probability of a firm being active if both firms are in R

Unique Symmetric Equilibrium: Both Firms in D

- it is optimal for a firm to be active with certainty if and only if:

$$(p_d(1 - p_d) + 0.5p_d^2)\gamma R - 1 \geq 0$$

$$\bar{R}_{dd}^{II} \equiv \frac{1}{p_d(1 - 0.5p_d)\gamma} \tag{1}$$

- At least one firm is active if and only if

$$p_d\gamma R - 1 \geq 0$$

$$\bar{R}_{dd}^{I} \equiv \frac{1}{p_d\gamma} \tag{2}$$

Unique Symmetric Equilibrium: Both Firms in D

Lemma

Suppose that both firms have qualified to the development stage; then in the unique symmetric equilibrium

$$\bar{q}_{dd} = \begin{cases} 0, & \text{if} \quad R \leq \bar{R}^{I}_{dd} \\ \frac{2}{p_d}\left(1 - \frac{\bar{R}^{I}_{dd}}{R}\right), & \text{if} \quad \bar{R}^{I}_{dd} \leq R \leq \bar{R}^{II}_{dd} \\ 1, & \text{otherwise} \end{cases} \tag{3}$$

The equilibrium value of each firm is given by

$$\bar{V}_{dd} = \begin{cases} 0, & \text{if} \quad R \leq \bar{R}^{II}_{dd} \\ \frac{\gamma}{2}\left(R - \bar{R}^{II}_{dd}\right), & \text{otherwise} \end{cases} \tag{4}$$

Unique Symmetric Equilibrium: One in R - One in D

- Given stationarity, the necessary and sufficient condition for the lagging firm to be active is

$$p_r(1-p_d)\bar{V}_{dd} - 1 \geq 0$$

$$R \geq \bar{R}_{rd} \equiv \bar{R}^{II}_{dd} + \frac{2}{p_r(1-p_d)\gamma} \tag{5}$$

- At least one firm is active if and only if

$$p_d\gamma R - 1 \geq 0$$

$$\bar{R}^{I}_{dd} \equiv \frac{1}{p_d\gamma} \tag{6}$$

Unique Symmetric Equilibrium: One in R - One in D

Lemma

Suppose that one firm is in the research stage and the other in the development stage; then in the unique symmetric equilibrium

$$\bar{q}_{rd} = \begin{cases} 0, & \textit{if } R \leq \bar{R}_{rd} \\ 1, & \textit{otherwise} \end{cases}, \bar{q}_{dr} = \begin{cases} 0, & \textit{if } R \leq \bar{R}_{d}^{l} \\ 1, & \textit{otherwise} \end{cases} \tag{7}$$

The equilibrium values of the two firms are respectively

$$\bar{V}_{rd} = \begin{cases} 0, & \textit{if } R \leq \bar{R}_{rd} \\ \frac{p_r(1-p_d)\bar{V}_{dd}-1}{p_d+p_r(1-p_d)}, & \textit{otherwise} \end{cases}, \bar{V}_{dr} = \begin{cases} 0, & \textit{if } R \leq \bar{R}_{dd}^{l} \\ V_{dr}^{1}, & \textit{if } R_{dd}^{l} \leq R \leq \bar{R}_{rd} \\ V_{dr}^{2}, & \textit{otherwise} \end{cases} \tag{8}$$

where

$$\bar{V}_{dr}^{1} \equiv \gamma(R - \bar{R}_{dd}^{l}), \bar{V}_{dr}^{2} \equiv \frac{p_d \gamma R - 1 + p_r(1-p_d)\bar{V}_{dd}}{p_d + p_r(1-p_d)}$$

Unique Symmetric Equilibrium: One in R - One in D

Lemma

(i) $\bar{V}_{dd}$ and $\bar{V}_{rd}$ are continuous and increasing in R with $\bar{V}_{dd}$ being strictly increasing for $R > \bar{R}^{II}_{dd}$ and $\bar{V}_{rd}$ for $R > \bar{R}_{rd}$, (ii) $\bar{V}_{dr}$ is continuous everywhere but exhibits a (downward) jump discontinuity at $R = R_{rd}$. It is strictly increasing in $[\bar{R}^{I}_{dd}, \bar{R}_{rd})$ and $(\bar{R}_{rd}, \infty)$. Moreover, for every $R \geq \bar{R}_{rd}$

$$\Delta\bar{V}_{dr}, \frac{\partial\Delta\bar{V}_{dr}}{\partial p_r}, \frac{\partial\Delta\bar{V}_{dr}}{\partial\gamma}, \frac{\partial\Delta\bar{V}_{dr}}{\partial R} > 0$$

Unique Symmetric Equilibrium: Both in R

- Following τ unsuccessful experiments, the expected payoff of experimenting in a period, provided that the rival does not experiment, is

$$\alpha_\tau p_r \bar{V}_{dr} - 1 \tag{9}$$

$$\bar{\alpha}^I = \min\left\{1, \frac{1}{p_r \bar{V}_{dr}}\right\} \tag{10}$$

Proposition

$\bar{\alpha}^I = 1$ for $R \leq \bar{R}^I_{rr}$ and $\bar{\alpha}^I < 1$ for $R > \bar{R}^I_{rr}$, where

$$\bar{R}^I_{rr} \equiv \bar{R}^I_{dd} + \frac{1}{\gamma p_r} \tag{11}$$

Moreover $\bar{\alpha}^I$ is continuous and strictly decreasing in $[\bar{R}^I_{rr}, \bar{R}_{rd})$ and $(\bar{R}_{rd}, \infty)$. It exhibits an (upwards) jump discontinuity at $R = \bar{R}_{rd}$.

Unique Symmetric Equilibrium: Both in R

- Following τ unsuccessful experiments, the expected payoff of continuing experimenting in this period, provided that the rival experiments, is given by:

$$\alpha_\tau(p_r(1-p_r)\bar{V}_{dr} + (1-p_r)p_r\bar{V}_{rd} + p_r^2\bar{V}_{dd}) - 1 \quad (12)$$

$$\bar{\alpha}^{II} = \min\left\{1, \frac{1}{p_r((1-p_r)(\bar{V}_{dr}+\bar{V}_{rd}) + p_r\bar{V}_{dd})}\right\} \quad (13)$$

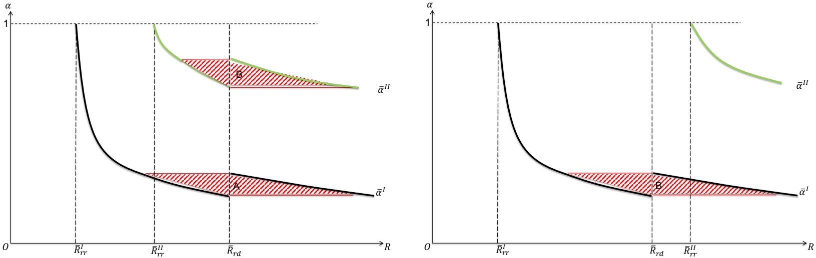
Proposition

If

$$\frac{p_r(1-p_r)}{2(1-0.5p_d)} + \frac{2-p_r}{1-p_d} > 1 \quad (14)$$

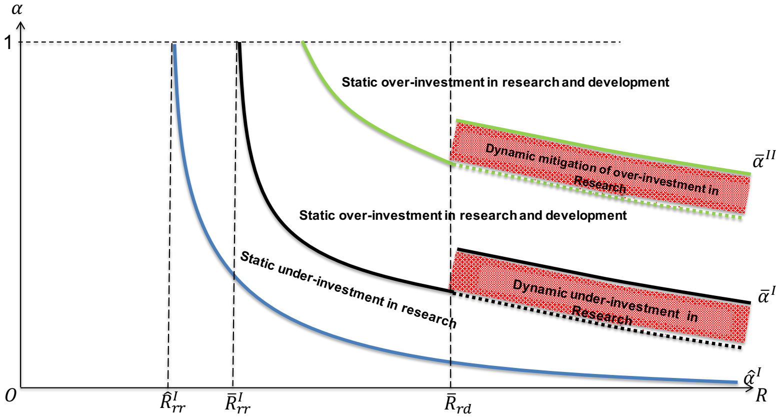
then, for some $\bar{R}^{II}_{dd} < \bar{R}^{II}_{rr} < \bar{R}_{rd}$*,* $\bar{\alpha}^{II} = 1$ *for* $R \leq \bar{R}^{II}_{rr}$*,* $\bar{\alpha}^{II} < 1$ *for* $\bar{R}^{II}_{rr} < R < \bar{R}_{rd}$ *and* $\bar{\alpha}^{II} \leq 1$ *for* $R > \bar{R}_{rd}$*. Moreover,* $\bar{\alpha}^{II}$ *is continuous in* $R \in [0, \bar{R}_{rd})$ *and* $R \in (\bar{R}_{rd}, \infty)$ *and exhibits an (upward) jump discontinuity at* $R = \bar{R}_{rd}$*.*
If (14) does not hold, then $\bar{\alpha}^{II}$ *is decreasing and continuous in* R *and strictly decreasing for some* R *sufficiently large.*

Unique Symmetric Equilibrium: Stopping times



(a) $\bar{R}^{II}_{rr} < \bar{R}_{rd}$ (b) $\bar{R}^{II}_{rr} > \bar{R}_{rd}$

Welfare



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1
O
Static over-investment in research and development
Dynamic mitigation of over-investment in Research
$\bar{\alpha}^{II}$
Static over-investment in research and development
Static under-investment in research
Dynamic under-investment in Research
$\bar{\alpha}^{I}$
$\hat{\alpha}^{I}$
$\hat{R}^{I}_{rr}$
$\bar{R}^{I}_{rr}$
$\bar{R}_{rd}$
R

Robustness

- Discounting: results hold even though the planners problem is more involved because there is an inter-temporal trade off
- Non-binary R&D investment: Less tractable
- Uncertain development stage
- Perfect imitation

Potential Extensions

- Asymmetric equilibria
- Asymmetric firms: one firm is stronger in R and the other in D
- Asymmetric info:
  - Firms privately observe the intermediate result and decide to disclose or not
  - Firms are asymmetrically informed about the cost of running experiments (e.g., $\{0, 1\}$)
- Optimal patent rules
- Protecting intermediate break-throughs (e.g., stringency of patent law)

Related Literature

- **One-stage (static) races**: Loury (1979), Lee and Wilde (1980), Dasgupta and Stiglitz (1980)
- **One-stage (dynamic) races**: Reinganum (1981,1982), Judd (2003), Doraszelski (2003), etc.
- **Multi-stage (dynamic) races**: Fudenberg et al. (1983), Harris and Vickers (1985,1987), Grossman and Shapiro (1987), Lippman and McCardle (1987), etc.
- **Learning**: Choi (1990), Malueg and Tsutsui (1997), etc.
- **Sequential innovation**: Scotchmer (1991, 1996), Scotchmer and Green (1990), Hopenhayn et al. (2006), Bessen and Maskin (2010), etc.
- **Strategic experimentation**: Bolton and Harris (1999), Keller et al. (2005), et al.